ECON 214
Elements of Statistics for Economists
2016/2017

Topic – The Normal Distribution

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Overview

- This lecture continues with our study of probability distributions by examining a very important continuous probability distribution: **the normal probability distribution**.
- As noted in the previous lecture, a continuous random variable is one that can assume an infinite number of possible values within a specified range.
- We shall examine the main characteristics of a normal probability distribution, examine the normal curve and the standard normal distribution, and then calculate probabilities for normally distributed variables.
- We shall also discuss how to determine percentile points for normally distributed variables and how the binomial and the Poisson distributions can be approximated with the normal distribution.

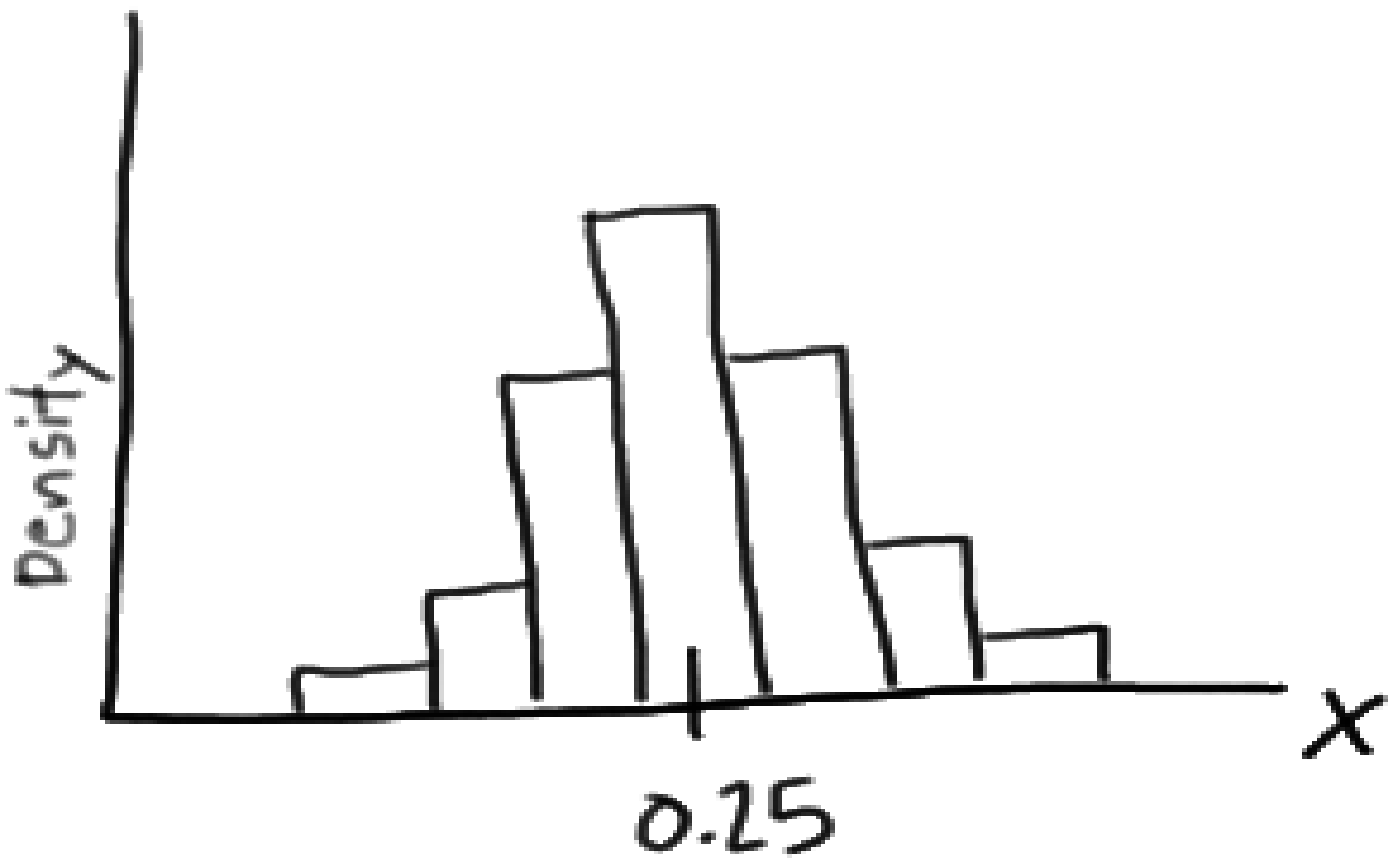
Overview cont'd

- At the end of this lecture, the student will
  - Be able to list the characteristics of the normal probability distribution.
  - Be able to define and calculate *z* values.
  - Be able to determine the probability that an observation will lie between two points using the standard normal distribution.
  - Be able to determine the probability that an observation will lie above (or below) a given value using the standard normal distribution.
  - Compute percentile points for normally distributed variables.
  - Use the normal distribution to approximate the binomial distribution.
  - Use the normal distribution to approximate the Poisson distribution.

Continuous Random Variables

- A continuous random variable is one that can assume an infinite number of possible values within a specified range.
- Consider for example a fast-food chain producing hamburgers.
- Assume samples of hamburger are taken and their weights measured (in kilograms).
- The probability (relative frequency) distribution of this continuous random variable (weight of hamburger) can be characterized graphically (by a histogram).

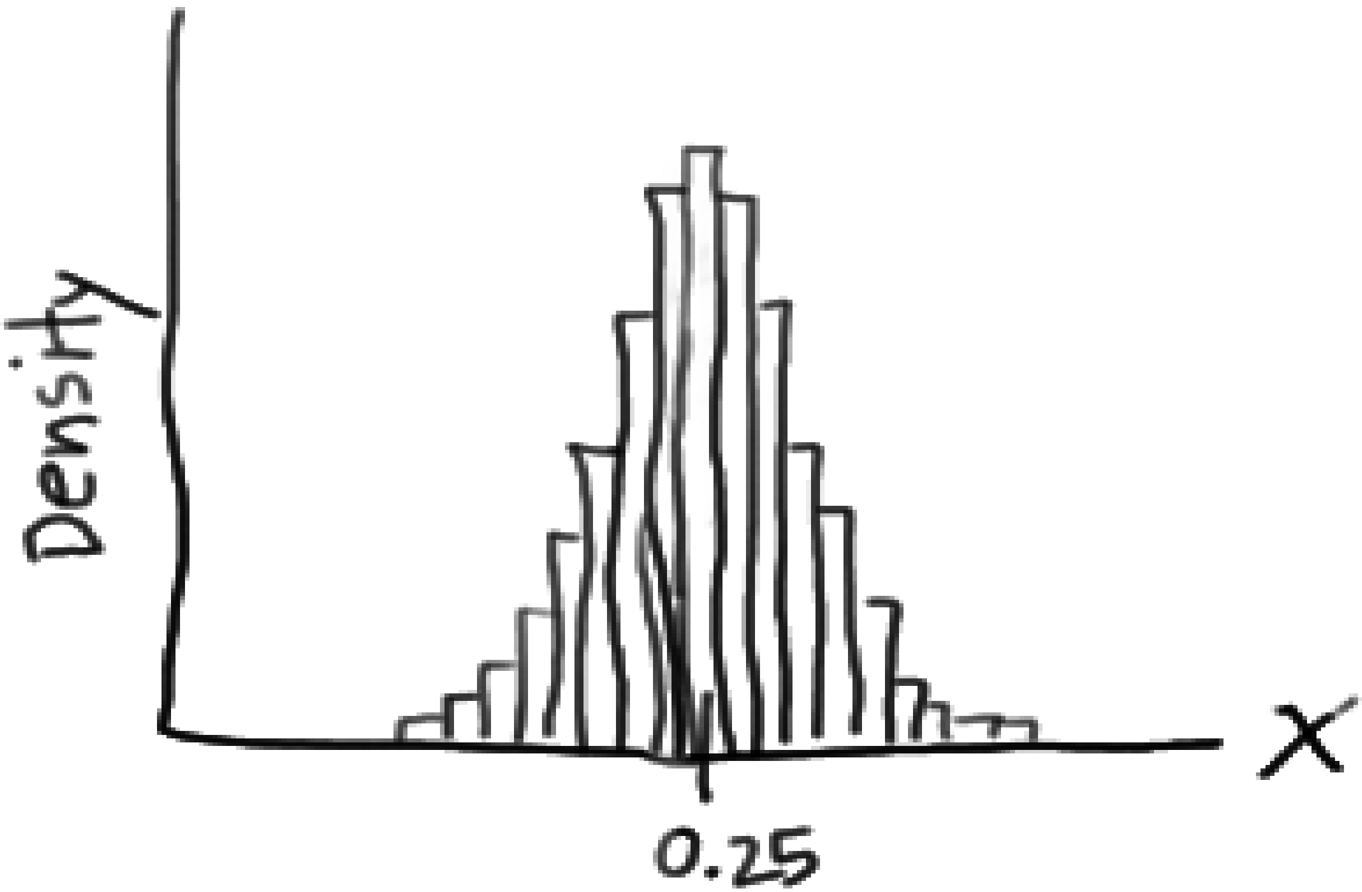
Continuous Random Variables



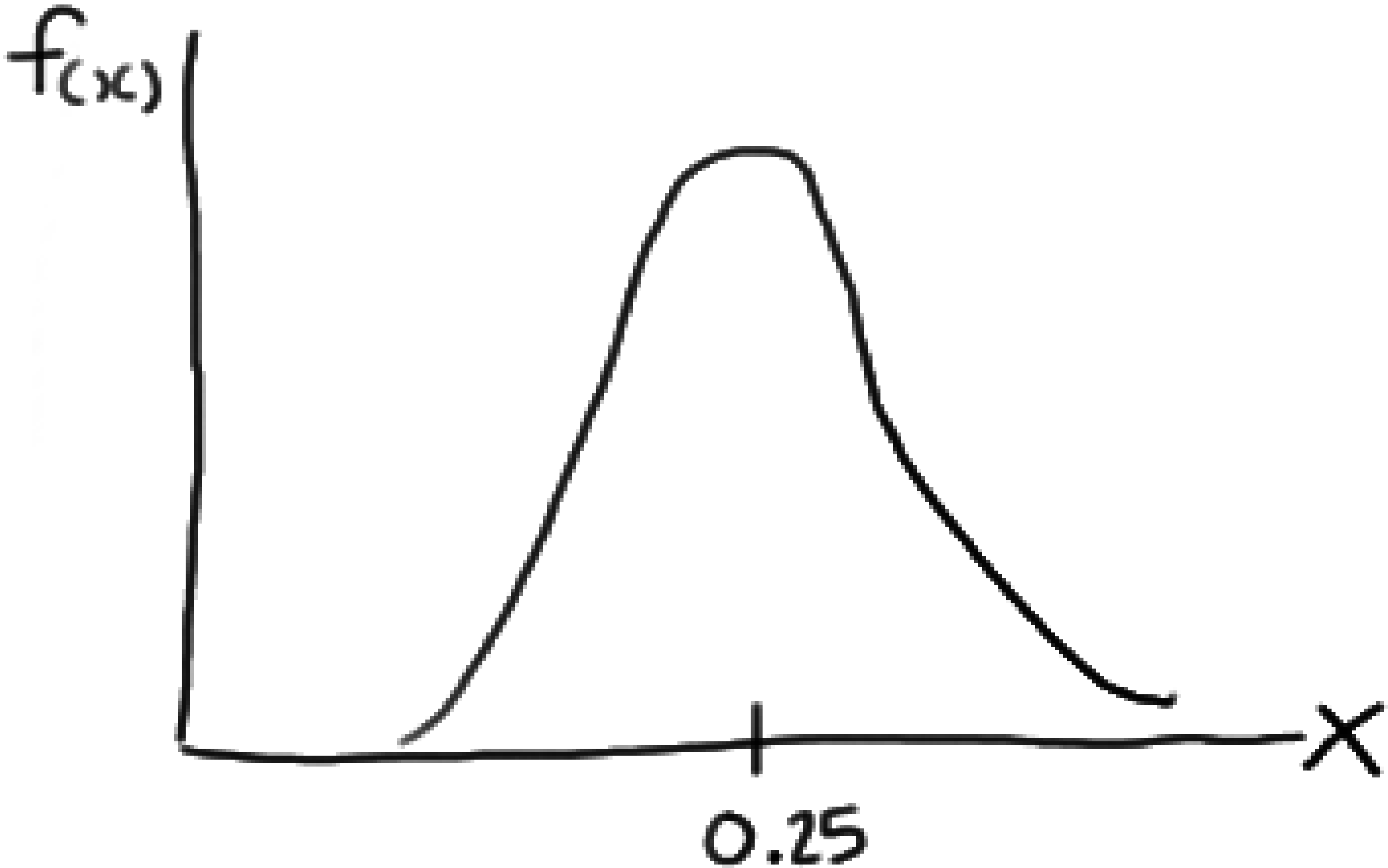
Continuous Random Variables

- The area of the bar representing each class interval is equal to the proportion of all the measurements (hamburgers) within each class (or weight category).
- The area under this histogram must equal 1 because the sum of the proportions in all the classes must equal 1.
- As the number of measurements becomes very large so that the classes become more numerous and the bars smaller the histogram can be approximated by a smooth curve.

Continuous Random Variables



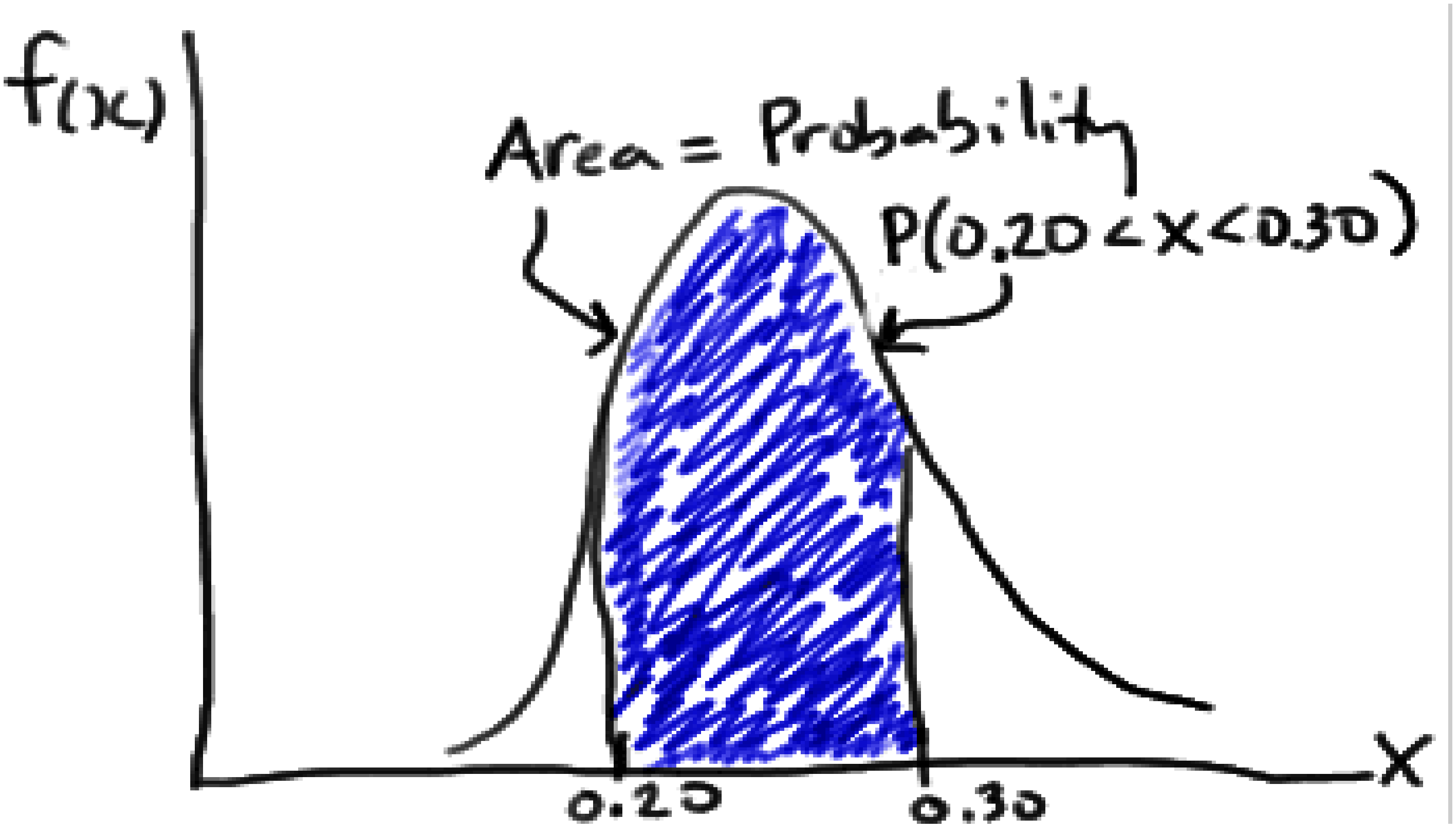
Continuous Random Variables



Continuous Random Variables

- The total area under this smooth curve equals 1.
- The proportion of measurements (weight of hamburgers) within a given range can be found by the area under the smooth curve over this range.
- This curve is important because we can use it to determine the probability that measurements (e.g. weight of a randomly selected hamburger) lie within a given range (such as between 0.20 and 0.30 kg.)

Continuous Random Variables



Continuous Random Variables

- The smooth curve so obtained is called a probability density function or probability curve.
- The total area under any probability density function must equal 1.
- The probability that the random variable will assume a value between any two points, from say, x_1 to x_2, equals the area under the curve from x_1 to $x_{2.}$
- Thus for continuous random variables we are interested in calculating probabilities over a range of values.
- Note then that the probability that a continuous random variable is precisely equal to a particular value is zero.

The Normal Probability Distribution

- The most important continuous probability distribution is the normal distribution.
- The formula for the probability density function of the normal random variable is

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}[(x-\mu)/\sigma]^2}$$

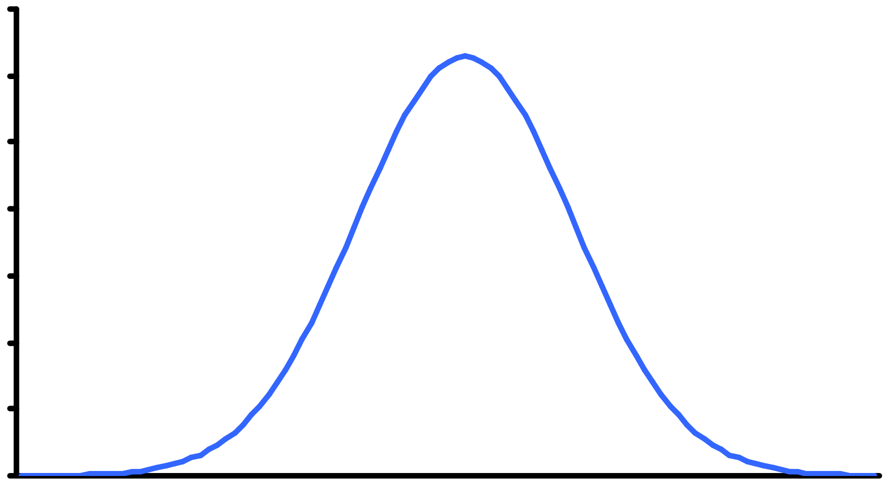
- Where ***X*** is said to have a normal distribution with mean **μ** and variance **σ^2**.

The Normal Distribution

- Examples of normally distributed variables:
  - IQ
  - Men's heights
  - Women's heights
  - The sample mean

The Normal Distribution

- The normal distribution has the following characteristics:
  - Bell shaped
  - Symmetric about the mean
  - Unimodal
  - The area under the curve is 100% = 1
  - Its shape and location depends on the mean and standard deviation
  - Extends from x = -∞ to + ∞ (in theory).

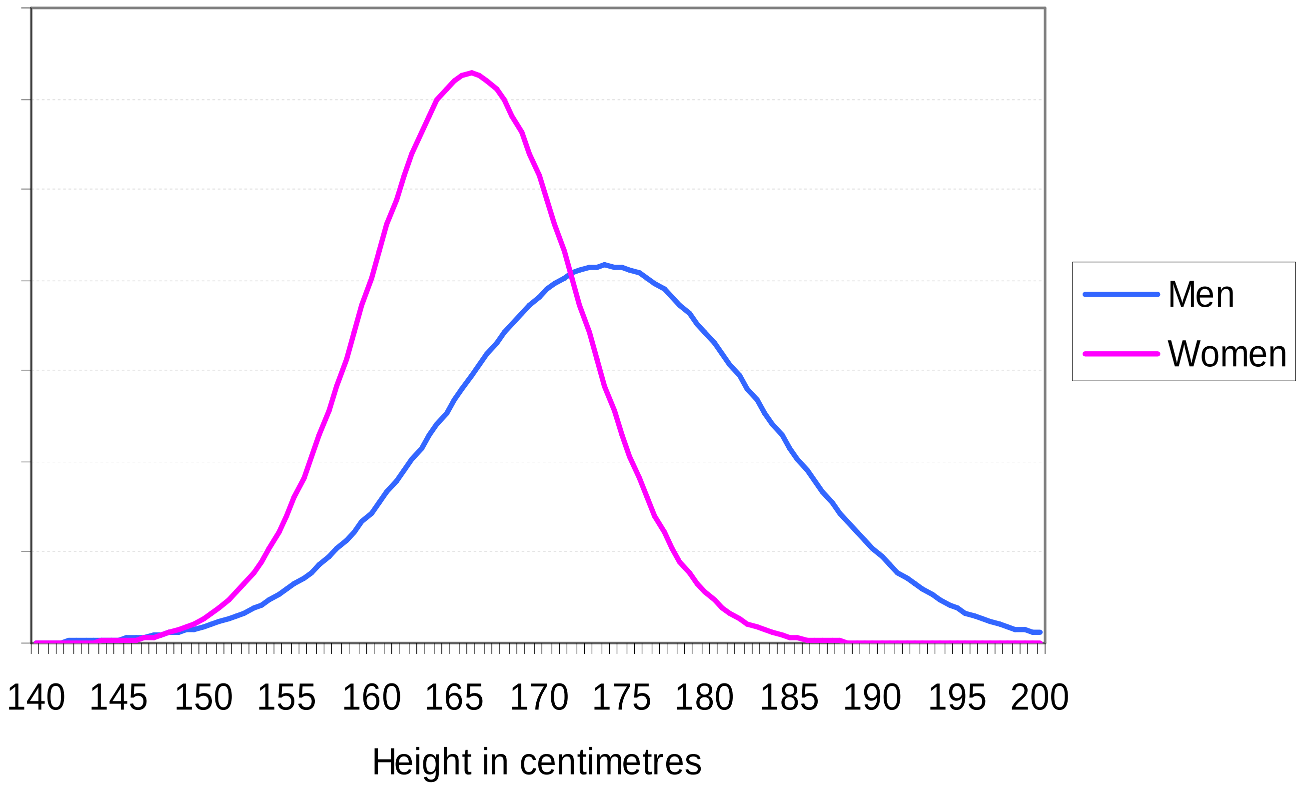


The Normal Distribution

- The two parameters of the Normal distribution are the mean μ and the variance σ^2.
  - $x \sim N(\mu, \sigma^2)$
- Men's heights are Normally distributed with mean 174 cm and variance 92.16 cm.
  - $x_M \sim N(174, 92.16)$
- Women's heights are Normally distributed with a mean of 166 cm and variance 40.32 cm.
  - $x_W \sim N(166, 40.32)$

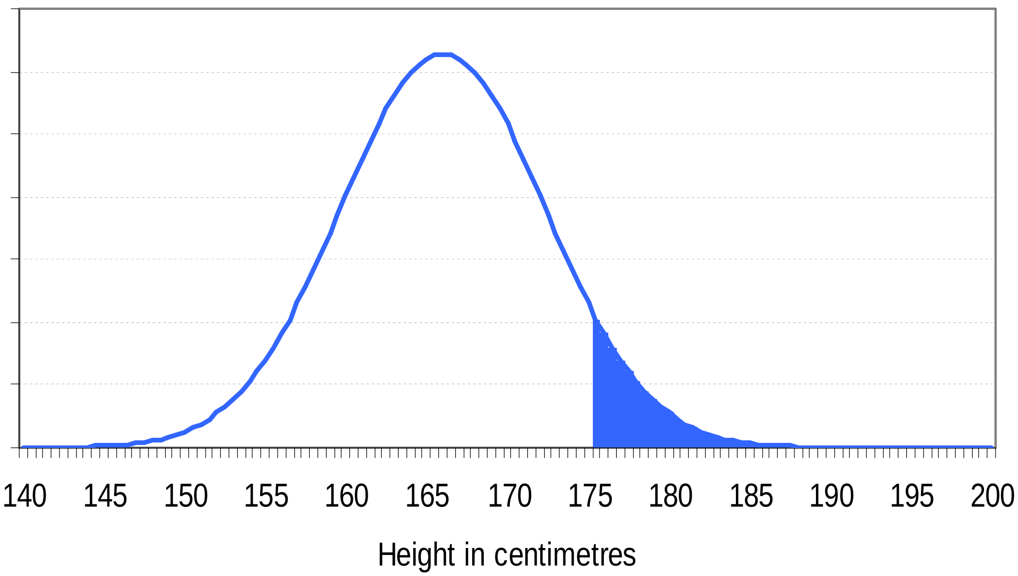
The Normal Distribution

- Graph of men's and women's heights



The Normal Distribution

- Areas under the distribution
  - We can determine from the normal distribution the proportion of measurements in a given range.
  - **Example**: What proportion of women are taller than 175 cm?
  - It is the (blue) shaded area.



The Normal Distribution

- There is not just one normal probability distribution but rather a family of curves.
- Depending on the values of its parameters (mean and standard deviation) the location and shape of the normal curve can vary considerably.

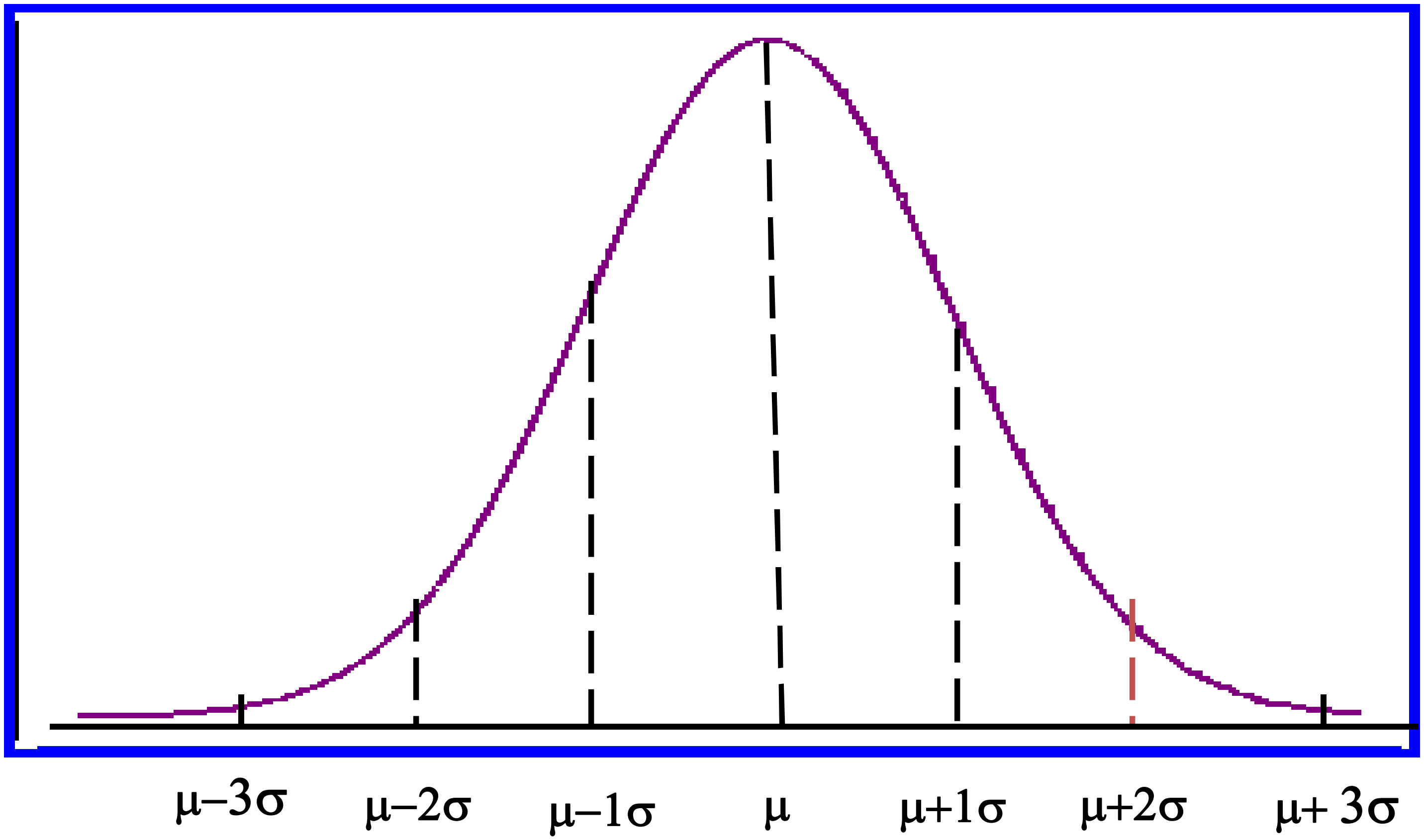
Importance of the Normal Distribution

- Measurements in many random processes are known to have distributions similar to the normal distribution.
- Normal probabilities can be used to approximate other probability distributions such as the Binomial and the Poisson.
- Distributions of certain sample statistics such as the sample mean are approximately normally distributed when the sample size is relatively large, a result called the Central Limit Theorem.

Importance of the Normal Distribution

- If a variable is normally distributed, it is always true that
  - 68.3% of observations will lie within one standard deviation of the mean, i.e. $X = \mu \pm \sigma$
  - 95.4% of observations will lie within two standard deviations of the mean, i.e. $X = \mu \pm 2\sigma$
  - 99.7% of observations will lie within three standard deviations of the mean, i.e. $X = \mu \pm 3\sigma$

The normal curve showing the relationship between σ and μ



Calculating probabilities using the standard normal

- Normal curves vary in shape because of differences in mean and standard deviation **(see slide 16)**.
- To calculate probabilities we need the normal curve (distribution) based on the particular values of **μ** and **σ**.
- However we can express any normal random variable as a deviation from its mean and measure these deviations in units of its standard deviation **(You will see an example soon)**.

Calculating probabilities using the standard normal

- That is, subtract the mean (**μ**) from the value of the normal random variable (X) and divide the result by the standard deviation (**σ**).
- The resulting variable, denoted ***Z***, is called a **standard normal** variable and its curve is called the standard normal curve.
- The distribution of any normal random variable will conform to the standard normal irrespective of the values for its mean and standard deviation.

Calculating probabilities using the standard normal

- If *X* is a normally distributed random variable, any value of *X* can be converted to the equivalent value, *Z*, for the standard normal distribution by the formula

$$Z = \frac{X - \mu}{\sigma}$$

- *Z* tells us the number of standard deviations the value of *X* is from the mean.
- The standard normal has a mean of zero and variance of 1, i.e., Z ~ N(0, 1).

Calculating probabilities using the standard normal

- Tables for normal probability values are based on one particular distribution: **the standard normal**; from which probability values can be read irrespective of the parameters **(i.e. mean and standard deviation)** of the distribution.
- **Example** - **following from the illustration on slide 17, c**onsider the height of women.
- How many standard deviations is a height of 175cm above the mean of 166cm?

Calculating probabilities using the standard normal

- We know that women's heights are Normally distributed with a mean of 166 cm and variance 40.32 cm.
- The standard deviation is √40.32 = 6.35, hence

$$Z = \frac{175 - 166}{6.35} = 1.42$$

- so 175 lies 1.42 standard deviations above the mean.
- How much of the Normal distribution lies beyond 1.42 standard deviations above the mean?
- We can read this from normal tables (normal tables are found at the Appendix of any standard statistics text).

Calculating probabilities using the standard normal

z	0.00	0.01	0.02	0.03	0.04	0.05	...
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	

Calculating probabilities using the standard normal

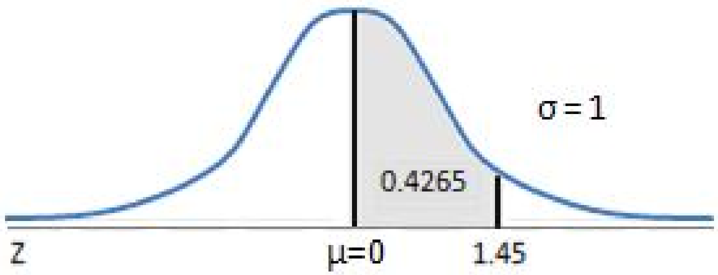
- The answer is .0778, meaning 7.78% of women are taller than 175 cm.
- What we have done in essence is to calculate the probability that the height of a randomly chosen woman is more than 175cm.
- That is, we want to find the area in the tail of the distribution (area under the curve) above (or to the right of) 175cm.
- To do this, we must first calculate the *Z*-score (or value) corresponding to 175cm, giving us the number of standard deviations between the mean and the desired height.
- We then look the *Z*-score up in tables. That's exactly what we have done!

Calculating probabilities using the standard normal

- Note that in this table the probabilities are read as the area under the curve to the right of the value of Z (for positive values of Z or starting from Z=0)
- There are other versions of the table so it is important to know how to read from a particular table.
- What is often referred to as **half table** is on the next slide.
- The area under the curve is read with zero as the reference point.

Areas Under the One-Tailed Standard Normal Curve

This table provides the area between the mean and some Z score.
For example, when Z score = 1.45 the area = 0.4265.



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545

Calculating probabilities using the standard normal

- In this half table, the probability values (area under the curve) are for *Z* values between zero (lower bound) and an upper bound *Z* value.
- In the previous example, we wanted the probability: $P(Z>1.42)$
- But the table only gives us $P(0<Z<1.42)$
- We can obtain our desired probability as $P(Z>1.42)= 0.5-P(0<Z<1.42)$ since for the half table, the total area under the curve is 0.5.

Calculating probabilities using the standard normal

- So we must read the area under the curve for Z=1.42 (upper bound).
- It is read from the extreme left (*Z*) column for 1.4 under 0.02 (the top-most row) and we have 0.4222.
- So $P(0<Z<1.42) = 0.4222$
- And therefore $P(Z>1.42)= 0.5-P(0<Z<1.42)$ $=0.5-0.4222=0.0778$ as before.

Calculating probabilities using the standard normal

- **Another example**: Suppose the time required to repair equipment by company maintenance personnel is normally distributed with mean of 50 minutes and standard deviation of 10 minutes. What is the probability that a randomly chosen equipment will require between 50 and 60 minutes to repair?
- Let X denote equipment repair time.
- We want to calculate the probability that X lies between 50 and 60.
- Or $P(50 \leq X \leq 60)$

Calculating probabilities using the standard normal

- Determine the Z values for 50 and 60.

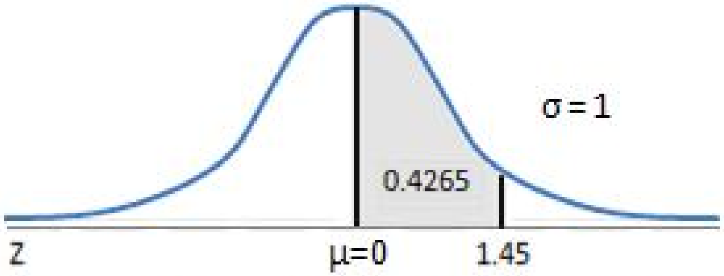
$$X = 50 \Rightarrow Z = \frac{X-\mu}{\sigma} = \frac{50-50}{10} = 0$$

$$X = 60 \Rightarrow Z = \frac{X-\mu}{\sigma} = \frac{60-50}{10} = 1.00$$

- So we have P(0 ≤ Z ≤ 1.00)
- We read off the area under the standard normal curve from zero to 1.00 from the normal table.

Areas Under the One-Tailed Standard Normal Curve

This table provides the area between the mean and some Z score.
For example, when Z score = 1.45 the area = 0.4265.



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
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0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
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0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
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1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
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1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545

Calculating probabilities using the standard normal

- The answer is **.3413** (read as 1.0 under 0.00).
- This was quite easy because the lower bound was at the mean (or zero).
- Most problems will not have the lower bound at the mean.
- Nevertheless the normal table can be used to calculate the relevant probabilities by the addition or subtraction of appropriate areas under the curve.
- **For instance**: Find the probability that more than 70 minutes will be required to repair the equipment.

Calculating probabilities using the standard normal

- We want P(X>70) = P[Z>(70-50)/10)=P(Z>2.00)
- Reading from the first table I introduced you to, P(Z>2.00) = 0.0228
- But using the half table we can write this as:

  $0.5 – P(0 \leq Z \leq 2) = 0.5 - 0.4772 = 0.0228$
- So depending on the type of table being used, we can calculate the required probability appropriately.
- **Henceforth we shall use the half table.**

Calculating probabilities using the standard normal

- **Example**: Find the probability that the equipment-repair time is between 35 and 50 minutes.
- We want $P(35 \leq X \leq 50)$
- Converting to Z values we have

  $P(-1.5 \leq Z \leq 0) \equiv P(0 \leq Z \leq 1.5) = .4332,$

  since the normal curve is symmetrical.

Calculating probabilities using the standard normal

- **Example**: Find the probability that the required equipment-repair time is between 40 and 70 minutes.
- $P(40 \leq X \leq 70) \equiv P(-1 \leq Z \leq 2) = P(-1 \leq Z \leq 0) + P(0 \leq Z \leq 2)$

  $= .3413 + .4772 = .8185$

Calculating probabilities using the standard normal

- **Example**: Find the probability that the required equipment-repair time is either less than 25 minutes or greater than 75 minutes.
- $P(X < 25)$ or $P(X > 75) = P(X < 25) + P(X > 75)$

$$= P(Z < -2.5) + P(Z > 2.5)$$

$$= [.5 – P(-2.5<Z<0)] + [.5 – P(0<Z<2.5)]$$

$$= 1 – 2 P(0<Z<2.5) = 1 – 2(0.4938)$$

$$= 1 - .9876 = .0124$$

Percentile points for normally distributed variables

- You discussed percentiles in under descriptive statistics.
- For example, the 90th percentile is the value *X* such that 90% of observations are below this value and 10% above it.
- In the case of the standard normal, the 90th percentile is the value *Z* such that the area under the normal curve to the left of this value (*Z*) is .9000 and the area to the right is .1000.

Percentile points for normally distributed variables

- To determine the value of a percentile point for any normally distributed variable, *X*, other than the standard normal, we first find the *Z* value for the percentile point and then convert it into its equivalent *X* value by solving for *X* from the formula

$$Z = \frac{X - \mu}{\sigma}$$

- Thus

$$X = \mu + Z\sigma$$

Percentile points for normally distributed variables

- For example, using the question on equipment-repair time, find the repair time at the 90th percentile.
- This implies we want to find the value of *Z* such that the area under the standard normal curve to the left of *Z* is .9000
- Given the standard normal table we are using (where the reading starts from the mean of zero), we must find the *Z* value corresponding to .4000 (since the left half of the curve is .5000).

Percentile points for normally distributed variables

- We do this by looking into the body of the normal table and to locate the value closest to .4000, which is .3997 (see table on next slide).
- The Z value corresponding to .3997 is 1.28 (1.2 under 0.08).
- Thus $Z = 1.28$
- Given $\mu = 50$ and $\sigma = 10$ from the question,

  $$X = \mu + Z\sigma = 50 + 1.28\,(10) = 62.8 \text{ minutes.}$$

- The interpretation is that 90 percent of the equipment will require 62.8 minutes or less to repair, while 10 percent will require more than 62.8 minutes to repair.

Areas Under the One-Tailed Standard Normal Curve

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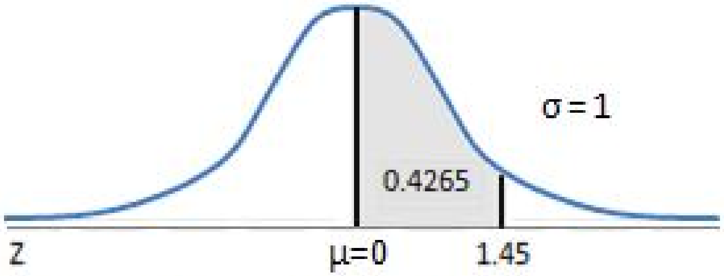
For example, when Z score = 1.45 the area = 0.4265.



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
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0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545

Percentile points for normally distributed variables

- Note that for percentiles less than 50, the value of *Z* will be negative since it will be to the left of the mean zero.
- For example, the 20th percentile repair time is ...

Percentile points for normally distributed variables

- The 20th percentile implies we want to find the value of *Z* such that the area under the standard normal curve to the left of *Z* is .2000 and the area to the right is .8000
- Using the half table, and given that the reading starts from zero, we must find the value corresponding to an area of .3000
- This gives .84 but because the percentile is less than 50, the *Z* value must be negative.
- Hence $Z = -.84$
- Therefore $X = \mu + Z\sigma = 50 + (-.84)(10) = 41.6$ minutes.

Normal approximation of the binomial distribution

- When $n > 30$ and $nP \geq 5$ we can approximate the Binomial with the normal
- Where
$$\mu = np$$
- And
$$\sigma = \sqrt{np(1-p)}$$
- We then apply the formula for calculating normal probabilities to determine the required probability.

Normal approximation of the binomial distribution

- When we approximate the Binomial with the normal, we are substituting a DPD for a CPD.
- Such substitution requires a correction for continuity.
- Suppose we wish to determine the probability of 20 or more heads in 30 tosses of a coin.

Normal approximation of the binomial distribution

- By the binomial we have

  $P(X \geq 20 / n = 30, P = .5) = .0494$

- Using normal approximation means

$$\mu = np = 30(.5) = 15$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{30(.5)(.5)} = 2.74$$

Normal approximation of the binomial distribution

- To determine the appropriate normal approximation, we need to interpret “20 or more” as if values on a continuous scale.
- Thus we must subtract .5 from 20 to get 19.5 since the lower bound of 20 starts from 19.5
- Hence

$$P_B(X \geq 20 / n = 30, P = .5) \approx P_N(X \geq 19.5 / \mu = 15, \sigma = 2.74)$$

- $P(X \geq 19.5) = P(Z \geq 1.64) = .0505$

Normal approximation of the binomial distribution

- When the Normal is used to approximate the Binomial, correction for continuity will always involve either adding .5 to or subtracting .5 from the number of successes specified.
- Generally the continuity correction is as follows:
- $P(X < b)$ => subtract .5 from *b* (*b* is exclusive)
- $P(X > a)$ => add .5 to *a* (*a* is exclusive)
- $P(X \leq b)$ => add .5 to *b* (*b* is inclusive)
- $P(X \geq a)$ => subtract .5 from *a* (*a* is inclusive)

Normal approximation of the Poisson distribution

- When the mean, **λ**, of the Poisson distribution is large, we can approximate with the normal.
- Approximation is appropriate when $\lambda \geq 10$
- Then

$$\mu = \lambda$$

$$\sigma = \sqrt{\lambda}$$

- The correction for continuity similarly applies.

Normal approximation of the Poisson distribution

- Suppose an average of 10 calls per day come through a telephone switchboard. What is the probability that 15 or more calls will come through on a randomly selected day.
- Using Poisson we obtain

$$P(X \geq 15 / \lambda = 10) = .0835$$

Normal approximation of the Poisson distribution

- Using normal approximation, then

  μ = λ=10 and σ = √ λ = 3.16

$$P_p(X \geq 15 / \lambda = 10) \simeq P_N(X \geq 14.5 / \mu = 10, \sigma = 3.16)$$

$$= P(Z \geq 1.42) = .0778$$